Since dU_1 may have an arbitrary value, we deduce that

$$\left(\frac{dR_1}{dU_1}\right)_{V_1} = \left(\frac{dR_2}{dU_2}\right)_{V_2}$$

Let

$$\varphi = \left(\frac{dR}{dU}\right)_V$$

Then we have

$$\varphi_1 = \varphi_2$$

A requirement for thermodynamic temperature is that in systems in equilibrium it have the same value. This requirement is met by $\varphi = \left(\frac{dR}{dU}\right)_V$. If it could be evaluated, it would have a fixed value for all systems in equilibrium, regardless of physical properties or the specific nature of the substance used to measure it (unlike the empirical temperature which depends on the particular thermometer).

For a reversible process

$$dR = \varphi dU + \left(\frac{dR}{dV}\right) dV$$

For the system under consideration

$$dU = dQ - pdV$$

$$dR = \varphi(dQ - pdV) + \left(\frac{dR}{dV}\right) dV$$

For an adiabatic reversible process we must have $dR = 0$; hence

$$\left(\frac{dR}{dV}\right) = \varphi p$$

$$dR = \varphi(dU + pdV) = \varphi dQ$$

If we consider now an irreversible process in which heat is transferred from one subsystem to another through a conductor of very small mass which makes negligible contribution to the thermodynamic properties, and in which all the irreversibility occurs, we have the following:

Change in subsystem 1

$$dR_1 = \varphi_1 dQ_1$$

Change in subsystem 2

$$dR_2 = \varphi_2 dQ_2$$

Overall, the process is adiabatic

$$dQ_2 + dQ_1 = 0$$

Hence

$$dR = dR_1 + dR_2 = (\varphi_1 - \varphi_2) dQ_1$$
$$dR > 0$$

Hence $(\varphi_1 - \varphi_2)$ and dQ_1 have the same sign. Heat is received by the system with the higher φ and leaves the system with the lower φ. To conform with traditional assignments of the concepts high and low to hot and cold, we will define thermodynamic temperature T by

$$T = 1/\varphi$$

If we now make the identification of R with entropy, we can summarize with

$$T = \left(\frac{dU}{dS}\right)_V$$

$$dU = TdS - pdV$$

Entropy is defined thus far with an unspecified scale and reference point. The scale is supplied by suitable definitions for heat units and for the temperature scale. The reference point remains arbitrary. The discussion of the thermodynamic temperature scale, and the techniques for making thermodynamic temperature observable, should follow here, but it is entirely conventional.

OTHER WORKS

While specific references to other discussions of the second law were omitted, claims for complete originality should not be inferred therefrom. In particular this discussion is inspired by the papers of Turner (*4*, *5*) and Buchdahl (*1*) and the textbook of Callen (*2*), while the present author accepts the responsibility for the way in which they are combined. The recent book by Giles (3) presents a complete axiomatic development along lines similar to those given here.

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2. Callen, Herbert B., "Thermodynamics," Wiley, New York (1960).
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5. *Ibid.*, **30**, 506 (1962).

On a Neglected Effect in Entrance Flow Analyses

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Dealy (*1*) has recently presented a discussion of analyses of the problem of laminar flow development in the entrance region of either a cylindrical tube or a parallel plane channel. He points out that at least twenty-five papers have appeared on the development of a parabolic profile from a uniform entry profile. He also points out that with only one exception these papers have all been based on equations of the boundary-layer type. His conclusions are that the boundary-layer analysis is invalid for small X and that the shape of the velocity profile is such as to make suspect stability analyses based on the boundary-layer equations. These conclusions are not questioned. However, the tenor of the note was entirely negative and it is possible that a reader could be led to the conclusion that only inaccurate results are obtained by use of the Prandtl boundary-layer equations. Such a conclusion would, for most purposes, be incorrect, since the boundary-layer equations yield results of remarkable accuracy except within one or two diameters or channel widths of the entrance. Some particular points which should be mentioned are:

1. The pressure drop and velocity distributions calculated from the boundary-layer solutions agree with all applicable published experimental measurements to within the accuracy of the

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Pressure drops and void fractions in horizontal two-phase flows of potassium, Smith, Lowell R., M. Rasin Tek, and Richard E. Balzhiser, **A.I.Ch.E. Journal, 12,** No. 1, p. 50 (January, 1966).

Key Words: Pressure Drops-8, Void Fractions-9, Flow-9, Horizontal-0, Two-Phase-0, Potassium-9, Sodium-9, Gamma-Ray Attenuation-10.

Abstract: Vapor-liquid flow of a potassium-8% sodium mixture was studied with the use of a boiling heat transfer test loop. Pressure drop data were obtained from an unheated test section. Void fractions were measured at the midpoint of the test section with gamma ray attenuation.

The low temperature removal of small quantities of nitrogen or methane from hydrogen gas by physical adsorption on a synthetic zeolite, Kidnay, Arthur J., and Michael J. Hiza, **A.I.Ch.E. Journal 12,** No. 1, p. 58 (January, 1966).

Key Words: A. Removal-8, Low Temperature-0, Nitrogen-3, Methane-3, Hydrogen-2, Adsorption-10,8,7, Zeolite-5, Pressure-6, Concentration-6, Time-6.

Abstract: A flow system was used to measure the physical adsorption isotherms of two mixtures of nitrogen and hydrogen and two mixtures of methane and hydrofen on a synthetic zeolite. Static systems were used to measure the pure component isotherms of nitrogen, methane, and hydrogen at the same temperature and over the appropriate pressure ranges.

Laminar converging flow of dilute polymer solutions in conical sections: Part I. Viscosity data, new viscosity model, tube flow solution, Sutterby, J. L., **A.I.Ch.E. Journal, 12,** No. 1, p. 63 (January, 1966).

Key Words: A. Rheology-8, Laminar Flow-8, Non-Newtonian-0, Visco-elastic-0, Nonviscometric-0, Converging-0, Conical Sections-5. B. Viscosity Data-8, Polymer Solutions-9, Natrosol-9, Hydroxyethyl Cellulose-9, Viscometers-10, Cone-and-Plate-0, Falling Sphere-0. C. Curve Fitting-8, Viscosity-9, Models-10. D. Flow Rate-6, Pressure Drop-7, Tubes-5.

Abstract: The converging flow investigation proper is described in Part II (*Trans. Soc Rheol.* 1965). In Part I, necessary background information is introduced. Viscosity data are presented for polymer solutions used in the converging flow experiment. These data are fitted with a new three-parameter non-Newtonian viscosity model which fits the data better than previous three-parameter models. The corresponding relationship between flow rate and pressure drop for laminar tube flow is derived.

Momentum, heat, and mass transfer in turbulent non-Newtonian boundary layers, Skelland, A. H. P., **A. I. Ch.E. Journal, 12,** No. 1, p. 69 (January, 1966).

Key Words: A. Momentum Transfer-8, Heat Transfer-8, Mass Transfer-8, Boundary Layers-9, Fluids-9, Non-Newtonian-0, Turbulent-0, Power Law Fluid-9, Velocity-8, Thickness-8, Shear Stress-8, Drag Force-8.

Abstract: The boundary-layer equations and a Blasius type of relationship between F and $N_{Re\,gen}$ are used to derive expressions for velocity distribution, local boundary-layer thickness, local shear stress, and total drag force for the turbulent boundary layer flow of a power law non-Newtonian fluid across a flat plate at zero incidence. Relationships are derived for the velocity at the edge of the laminar sublayer and for the thickness of the laminar sublayer.

An analogy between heat and momentum transfer is then used to obtain expressions for local and mean values of the heat transfer coefficient in a turbulent thermal boundary layer for power law materials flowing over flat plates. Analogous extensions to mass transfer are indicated.

A tentative criterion is suggested for characterizing the transition from laminar to turbulent boundary-layer flow of power law fluids.

Relationships combining the effects of a part-laminar part-turbulent boundary layer are presented.

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measurements. A recent summary and comparison is given by Sparrow, Lin, and Lundgren (2).

2. Entrance flow phenomena are particularly important in cooling of high Prandtl number fluids where viscosity is strongly temperature dependent. It has recently been shown that in such cases, solutions of the boundary-layer equations agree with experiments to within the accuracy of the measurements (3).

3. Wang and Longwell (4) have shown the deviations between the boundary-layer type solutions and their solutions of the Navier-Stokes equations to be important for very small values of X. Yet for the Reynolds number they studied, 300, the deviations from the boundary-layer solutions are of little consequence for values of X greater than about two channel widths. Specifically, the overall pressure drop calculated from the boundary-layer equations at an X distance of two channel widths deviates from the calculations of Wang and Longwell by about 10%, and Wang and Longwell feel the error in their work accounts for at least half of this difference. The entry length computed by Wang and Longwell agreed with boundary-layer predictions to within 2%.

In conclusion, it should be stated that the boundary-layer equations yield results which, in principle, are valid for the asymptote of large distance from the entrance and large Reynolds number. All evidence indicates that the asymptote is approached rapidly, so that for many purposes the error involved in using the boundary-layer equations rather than the complete Navier-Stokes equation is negligible.

It should be mentioned again that there is disagreement with Professor Dealy only in a matter of emphasis. Some of the most important research work in fluid mechanics is involved with more elaborate models than the boundary-layer equations. On the other hand, it is misleading to emphasize that the equations fail for "small" X without some quantitative discussion of "small." The parametric description of more sophisticated models is sufficiently cumbersome to assure us that the boundary-layer approach will continue to be of great value.

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2. Sparrow, E. M., S. H. Lin, and T. W. Lundgren, *Phys. Fluids*, **7**, 338 (1964).
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